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Quantization of Poisson pairs: the R -matrix approach

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We suggest an approach to the quantization problem of two compatible Poisson brackets in the case when one of them is associated with a solution of the classical Yang–Baxter equation. We show that the quantization scheme (a Poisson bracket \rightarrow an associate algebra, quantizing this bracket in the spirit of the Berezin–Lichnerovitch deformation quantization \rightarrow its representation in a Hilbert space) has to be enlarged. We represent the deformation algebras, quantizing the “ R -matrix” brackets, in a space with an S -symmetric pairing, where S is a solution of the corresponding quantum Yang–Baxter equation. An example of quantization of an “exotic” harmonic oscillator is discussed.

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An R -matrix is an invertible operator $R : V^{\otimes 2} \rightarrow V^{\otimes 2}$ satisfying the quantum Yang–Baxter equation (QYBE)

$$R^{12}R^{13}R^{23} = R^{23}R^{13}R^{12}.$$

Here $R^{12} = R \otimes \text{id}$ etc. and V is a finite- (or infinite-)dimensional linear space over the field $k = \mathbb{R}$ or \mathbb{C} . If $R = \sigma \cdot S$, where σ is the permutation $\sigma(v_1 \otimes v_2) = v_2 \otimes v_1$, $v_i \in V$, then QYBE can be rewritten as follows:

$$(S \otimes \text{id})(\text{id} \otimes S)(S \otimes \text{id}) = (\text{id} \otimes S)(S \otimes \text{id})(\text{id} \otimes S). \quad (0.1)$$

Thus, an R -matrix in the form (0.1) gives a “local” representation of the braid group. Henceforth we call an operator S satisfying (0.1) a YB operator. And we call a YB operator a symmetry if it satisfies the “unitarity” condition $S^2 = \text{id}$ (or $R^{12} \cdot R^{21} = \text{id}$, where $R^{21} = \sigma R^{12} \sigma$ for R).

A classical analogue of this object is a “classical R -matrix”, i.e. an operator $R : V^{\otimes 2} \rightarrow V^{\otimes 2}$ satisfying the equation